The present memoir (in the composition of which I have been assisted by a correspondence with Dr. Salmon) contains a further development of the theory of the skew surfaces generated by a line which meets a given curve or curves: viz. I consider,—1st, the surface generated by a line which meets each of three given curves of the orders m, n, p respectively; 2nd, the surface generated by a line which meets a given curve of the order m twice, and a given curve of the order n once; 3rd, the surface which meets a given curve of the order m three times; or, as it is very convenient to express it, I consider the skew surfaces, or say the "scrolls," $S(m, n, p)$, $S(m^2, n)$, $S(m^3)$. The chief results are embodied in the Table given after this introduction, at the commencement of the memoir. It is to be noticed that I attend throughout to the general theory, not considering otherwise than incidentally the effect of any singularity in the system of the given curves, or in the given curves separately: the memoir contains, however, some remarks as to what are the singularities material to a complete theory; and in particular as regards the surface $S(m^3)$. I am thus led to mention an entirely new kind of singularity of a curve in space—viz., such a curve has in general a determinate number of "lines through four points" (lines which meet the curve in four points); it may happen that of the lines through three points, which can be drawn through any point whatever of the curve, a certain number will unite together and form a line through four (or more) points, the number of the lines through four points (or through a greater number of points) so becoming infinite.

II. "Researches on the Refraction, Dispersion, and Sensitiveness of Liquids." By J. H. Gladstone, Ph.D., F.R.S., and the Rev. T. P. Dale, M.A., F.R.A.S. Received February 5, 1863.

(Abstract.)

This communication contains the results of some inquiries which were started by the authors in a previous paper "On the Influence of Temperature on the Refraction of Light"*. The same apparatus

* Phil. Trans. 1858, p. 887.

had been employed, but some modifications were introduced in the method of observation, which are described; and the amount of probable error from different sources was determined. The liquids experimented on were either prepared or purified in Dr. Gladstone's laboratory, or were specimens reputed to be pure, and lent for the purpose of this inquiry by Prof. Hofmann, Prof. Williamson, Prof. Frankland, Drs. Warren De la Rue and Hugo Müller, Mr. Buckton, Dr. Odling, Mr. A. H. Church, Mr. C. Greville Williams, and Mr. Piesse. The data are collected in two long tables forming two appendices: the first containing the refractive indices of the lines A, D, and H, of 78 specimens at two or three different temperatures; the second, the refractive indices of all the more important lines for 61 of these liquids, and 10 others at the temperature of the room when the observations were made.

Five points were investigated, and the following are the results arrived at with respect to each point.

I. *The relation between the change of refraction (sensitiveness) and the change of volume by heat.*—The uniform testimony of about 90 different liquids examined was that both refraction and dispersion diminish as the temperature increases.

The following Table will suffice as an example, showing as it does that the different rays are more sensitive in the order of their refrangibility:—

Liquid.	Temp.	Refractive Indices.						
		A.	B.	D.	E.	F.	G.	H.
Bisulphide of Carbon......	11° C.	1·6142	1·6207	1·6333	1·6465	1·6584	1·6836	1·7090
	36°·5	1·5945	1·6004	1·6120	1·6248	1·6362	1·6600	1·6827
Difference	....	0·0197	0·0203	0·0213	0·0217	0·0222	0·0236	0·0263

This change of refraction by heat was compared with the known or ascertained change of volume in bisulphide of carbon, water, methylic, ethylic, and amylic alcohols, ether, acetone, acetic acid, formic, acetic, and butyric ethers, methylic and ethylic iodides, salicylate of methyl, bromoform, benzole, xylole, cumole, nitrobenzole, hydrate of phenyl, the rectified oils of turpentine and Portugal and eugenic acid, and in every case it was found that the refractive index minus unity, multiplied by the volume, gave very nearly a

constant at different temperatures. Now every refractive index contains at least two coefficients: the one of refraction, which is represented by the theoretical limit of the spectrum; the other of dispersion, for which the difference between the refractive indices of H and A may be taken as the exponent. The refractive index, minus unity ($\mu - 1$), is termed by the authors the "refractive energy" of the substance, and this multiplied by the volume ($\mu - 1$), or divided by the density, is termed the "specific refractive energy." It was not found as a rule that the theoretical limit of the spectrum gave more truly a constant than the line A; but the difference is within experimental errors. The empirical law was therefore expressed as follows:—The refractive energy of a liquid varies directly with its density under the influence of change of temperature, or, in other words, the specific refractive energy of a liquid is a constant not affected by temperature. Yet the influence of dispersion renders this not absolutely accurate in the observed numbers, for the change of dispersion does not follow the same law, the spectrum contracting in some cases much more, and in other cases much less rapidly than the volume increases; indeed no relation is as yet discoverable between the change of dispersion and that of density.

II. *The refraction and dispersion of mixtures of liquids.*—This question has engaged the attention of several experimenters, only one of whom, however, M. Hoek, has offered a solution. His formula depends on $\mu^2 - 1$. Yet most of the results recorded were equally well explained on the supposition that the specific refractive energy of a mixture is the mean of the specific refractive energies of its components. It was clearly desirable to test this in some cases where the refractive indices of the liquids mixed were very wide apart. Fortunately, bisulphide of carbon and ether, substances almost at the opposite limits of the scale, were found to mix without condensation; and another good experiment was obtained with aniline and alcohol, on mixing which, however, some diminution of volume occurs. In both these cases the experimental numbers were slightly below those deduced from the mean of the specific refractive energies, the discrepancy being beyond the limits of probable error; yet no other formula could be devised which would give a nearer approximation to the indices actually observed.

III. *The refraction, dispersion, and sensitiveness of different*

members of homologous series.—Many such series were examined, and the results are tabulated, the refractive index of A and the length of the spectrum or dispersion being reduced, if necessary, to 20° C., and the sensitiveness being taken for the 10 degrees rising above 20° C.; the specific refractive energy, dispersion, and sensitiveness also form part of the Tables. Methylic, ethylic, amylic, and caprylic alcohols are the first series examined, and it is found that on ascending the series the refraction increases; the dispersion does so still more rapidly, while the sensitiveness remains nearly the same. Other homologous series of the same group, such as the iodides, compound ethers, or mercury compounds, were also examined, and they all agree in exhibiting a progressive change in refraction and dispersion with the advancing members of the series; but in which direction and to what extent depend on the other substances with which the compound radical is combined. Yet, if we regard not the actual indices, but these, minus unity, divided by the density, a pretty regular increase is found to take place as the series advance. The following Tables exhibit this:—

Specific Refractive Energy.

Radical.	Formula.	Alcohol.	Iodide.	Ether of Acid.	Formiate.	Acetate.	Butyrate.	Oxalate.	Mercury Compound.	Stannic Compound.	Hydride.
Methyl	$C_2 H_3$	·4105	·2359	·3905	...	·389	...	...	·1707	·3727	
Ethyl	$C_4 H_5$	·4482	·2614	·4127	·3905	·4127	·4402	·3502	·2112	·3876	
Propyl	$C_6 H_7$	...	...	·4333							
Butyl	$C_8 H_9$	...	...	·4402							
Amyl.........	$C_{10} H_{11}$	·4895	·3213	·4492	·4432	·4527	·4724	·4306			
Œnanthyl...	$C_{14} H_{15}$	...	...	·4750	...	...	...	...	...	...	·5499
Capryl	$C_{16} H_{17}$	·5096	...	...	...	...	...	...	...	...	·5522
Laurostearyl	$C_{14} H_{25}$	...	...	·4890							

Specific Dispersion.

Radical.	Alcohol.	Iodide.	Ether of Acid.	Acetate.	Mercury Compd.	Stannic Compd.	Hydride.
Methyl............	163	209	168	...	140	256	
Ethyl	190	218	174	174	170	268	
Propyl	...	...	191				
Butyl	...	...	191				
Amyl...............	212	224	198	198			
Œnanthyl.........	...	...	...	...	...	...	241
Capryl	237	...	...	...	...	...	237

Other groups of homologous bodies were also examined. Benzole, toluole, xylole, cumole, and cymole gave nearly the same numbers, and no regular progression. Pyridine, picoline, lutidine, and collidine showed an augmentation of the specific refractive energy, but a diminution of the specific dispersion with the advancing series. Chinoline and lepidine (which proved to be the most refractive organic liquid known) showed an increase of each of the optical properties by the addition of $C_2 H_2$. Thus the influence of the added increment on the rays of light differs in different groups, just as it does in respect to the boiling-point.

IV. *The refraction, dispersion, and sensitiveness of isomeric liquids.*—Several of the liquids, isomeric with the different members of the benzole series, were examined; some proved to be identical in all optical properties; others sensibly the same in actual refraction and dispersion, though slightly different in density; some again identical in density, but differing in optical properties; while other isomeric bodies differed slightly in each of these respects. Several hydrocarbons of the type $C_{20} H_{16}$, from essential oils, seemed to be identical in actual refraction, notwithstanding slight differences of their density. In dispersion, too, there were some variations; but not in sensitiveness. Other hydrocarbons, however, of the same ultimate composition, but differing considerably in physical properties, differed also optically. Compound ethers, as valerianic ether and acetate of amyl, which contain the same number of carbon, hydrogen, and oxygen elements, though differently arranged, are optically identical, as was partially shown by Delffs some years ago. Aniline and picoline, each empirically $C_{12} H_7 N$, are totally different. The conclusion arrived at is that isomeric bodies are sometimes widely different in these optical properties; but that in many cases, especially where there is close chemical relationship, there is identity also in this respect.

V. *The effect of chemical substitution.*—By observing the amount of change in the optical properties which results from a replacement of one element by another, the chemical type remaining the same, it seemed possible to arrive at a knowledge of the influence of the individual elements on the rays of light transmitted by them. Of the immense number of data required for the perfecting of such an inquiry, the following are afforded by the experiments already made.

The replacement of hydrogen by a compound radical, aniline—amyl-aniline; and water, alcohol, ether (according to Williamson's theory). Of hydrogen by oxygen—alcohol, acetic acid; ether, acetic ether; and carvene, carvole, eugenic acid. Of hydrogen by peroxide of nitrogen—benzole, nitrobenzole, dinitrobenzole (in solution); glycerine, nitroglycerine; and amylic alcohol, nitrate of amyl. Of hydrogen by chlorine—benzole, chlorobenzole, terchlorobenzole; and the substitution of chlorine by bromine—terchloride of phosphorus, terbromide of phosphorus; chloroform, bromoform; and bichloride of chlor-ethylene, bibromide of chlor-ethylene, bibromide of brom-ethylene. When hydrogen is replaced by some other body, there is generally an increase of the actual refraction and dispersion; but this is due to the increased weight, hydrogen having a very low actual, but a very high specific influence on the rays of light. In each of the five instances of two substitution-products, as, for instance, chlorobenzole and trichlorobenzole, the lower one always retains in its optical properties an intermediate position between the original substance and the higher product.

These experiments on substitution sufficed to show, as the examination of isomeric bodies had done, that the special influence exerted on the rays of light by the elements of a compound is greatly dependent on the manner of their combination.

The following is given as a generalization approximately, if not absolutely true:—Every liquid has a specific refractive energy composed of the specific refractive energies of its component elements, modified by the manner of combination, and which is unaffected by change of temperature, and this refractive energy accompanies it when mixed with other liquids.

III. "On the Change of Form assumed by Wrought Iron and other Metals when Heated and then Cooled by partial immersion in Water." By Lieut.-Col. H. Clerk, R.A., F.R.S. Received February 9, 1863.

Origin of the Experiments.—A short time ago, when about to shoe a wheel with a hoop-tire, to which it was necessary to give a bevel of about $\frac{3}{8}$ths of an inch, one of the workmen employed suggested that the bevel could be given by heating the tire red-hot and